

# Model Development for the Optimal Water Systems Planning

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## Abstract

A systems engineering approach is proposed for the optimal water supply chain management. The work introduces the idea of optimally allocating the existing resources quantifying the profitability of water use and eliminating inefficiencies rather than continuously seeking ways to expand the supply sources.

The developed mathematical model takes into account the costs of each supply source and the benefits from the water allocation to each user, allowing the water availability to be less than the total demand. The variables of the model include the time varying water quantities supplied by different water sources and the time varying water quantities being delivered to various users. Model constraints include demands, limitations in the capacity of the various water sources and technical specifications that must be followed in the water allocation. Optimisation criteria for the water planning are proposed aiming to the identification of the most efficient operation of the integrated water system.

**Keywords:** Mathematical Modeling and Optimisation, Water Allocation, Water Systems Optimisation

## 1. Introduction – Optimisation methods in Water Systems Planning

Water is the most valuable natural resource and its shortage is a serious problem being faced by many areas of the planet. The water supply chain management and optimisation are evolving as the most difficult and urgent problems [1]. Furthermore, in many areas, economic growth often brings expansion of various water demands and, thus, causes serious water shortages, either periodically or permanently. In this case, the efficiency of water allocation (i.e. what quantities are delivered to each user group) depends highly on the time period, since the existing water availability may not cover all the demands and priorities may need to be assigned to each user.

In remote areas, water is normally supplied from various local sources, such as dams, desalination plants and ground reservoirs, or even may be transferred by boats. On the other hand, various users impose conflicting demands on the resources, requiring water quantities that may temporarily not be available. Therefore, the problem of the optimal water system design and planning is created.

Several methodologies from systems engineering have increasingly been used over the last few decades for the design and operation of water resource systems. Optimisation models have also been applied for the solution of a number of problems related to the optimal planning of supply sources, or dealing with the total water resources management system [2-4]. However, limited research work has been carried out for the most difficult and urgent problem of the integrated water supply chain optimisation.

The present work approaches the water systems planning problem taking into account the characteristics of both, supplies and demands. The methodology and the

optimisation model proposed are generic and can be applied in any water system. For illustration purposes, special emphasis is given in the area of Aegean islands.

## **2. Overview of the water resources management problem in Hellenic Aegean Sea**

### *2.1. Water demand*

Water is a constrained resource in many areas of the planet. Most of the Aegean islands suffer from severe lack of good quality fresh water, mainly because of the low precipitation and the specific geomorphology of the islands. Furthermore, these places accept many tourists; especially during the summer period their population may be five times more than the winter, thus resulting in more serious and acute water shortage problems.

Classification of the water consumption includes urban users (commercial, permanent and seasonal domestic users), industrial and agricultural users. However, in the area under discussion, water demand originates from the agricultural and the urban users. Industry is not a significant water consumer in the islands.

### *2.2. Water supply*

The most common water supply sources in remote areas with limited water resources are the ground reservoirs and dams – associated with water treatment plants, the desalination plants, wells and boreholes, the transfer of water through ships, water recycle and reuse (not commonly used yet). For a long time, increasing water demand was covered by water transfer through ships in the Hellenic islands. However, there are very significant economic costs associated with this method, as well as the faith that it is completely unsustainable and does not create any infrastructure for the long-term problem solution.

The resulting costs of water are different for each of the above sources. In practice, the cost includes a fixed term, associated to the depreciation of the capital investment and a variable cost term. The desalted water has a significant operating cost, while the water from ground reservoirs and dams has a serious fixed cost term, because of the high capital investment required. Indicatively, desalted water cost accounts for 3 €/m<sup>3</sup>, water transferred by ships almost 7€/m<sup>3</sup> and from ground reservoirs almost 4,5 €/m<sup>3</sup>.

## **3. The Proposed Mathematical Model**

### *3.1. Basic Characteristics and Structure of the Proposed Model*

The mathematical model that is proposed in the present work intends to identify the optimal solution in the operation of the water system, taking into account:

- Various supply sources, each one with an associated water cost and a certain and possibly time varying capacity.
- Various users, each one associated with a time varying demand and a benefit for the use of water (expressed as a monetary value per cubic meter of water).

The objective of the model is the identification of the appropriate flow values from each supply source and of the quantities allocated to each user, keeping in mind that the total water availability may be less than the total demand. The allocation of the available water quantities will be made following the more sustainable principle that the real and most urgent needs must be satisfied [5]. At the same time, constant inefficiencies of the water system will be identified. Figure 1 shows a schematic representation of the system.

The supply sources provide water in a real or virtual storage tank; the storage tank has a specific capacity (upper limit) and a low limit that should never be violated. The usual supply sources taken into account in the present work are the desalination units, the ground Reservoirs and dams, the water transfer by ship, possible own water resources (e.g. wells). In fact the model can accommodate any type of water supply. The information that is required is its cost, capacity and any existing operational constraints.

3.2. Water Supply

The supply limits are determined from the capacity of each specific source. For the desalted water, the supply limit is the desalination unit’s capacity, for the ground reservoir is its capacity – taking also into account the capacity of the water treatment plant, since in most cases the water from a reservoir needs to be treated before reaching the consumer. The ship transfers water into the storage tank at specific time periods. The capacity limit in this case is determined by the quantity that has been transferred by the boat. The supply costs may simply be considered as linear terms multiplying the corresponding water quantity or follow more complicated economic functions. More specifically, the desalted water cost may be calculated as the sum of a fixed term, expressing the depreciation of the unit and a variable cost term or be expressed with a more complicated economic function, taking also into account various parameters of the unit’s operation [6]; the same is valid for the ground reservoir and the dam. On the contrary, the water transferred by ships has only a rather high variable cost term.

3.3. Water allocation

The users take water from the storage tank. The upper limits of the quantities being delivered to the various users are the corresponding time-varying demands. The water users are the agriculture (irrigation), the urban use (including permanent and seasonal domestic use and commercial use), the industry and other secondary uses. In case the required water quantity exceeds the available one, not all the requirements will be satisfied. This will definitely cause some consequences to the users (e.g. cancellation or limitation of expansion plans, losses etc.).

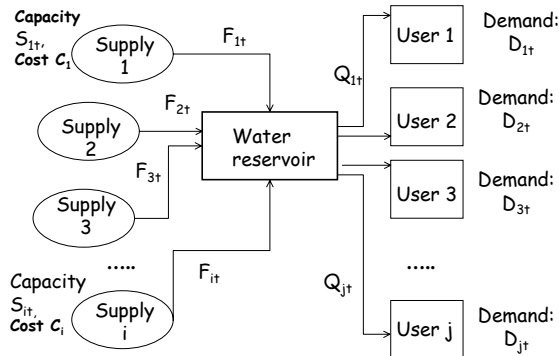


Figure 1 Schematic representation of the water system

The allocation of water to users will be determined by the optimisation. The model will allow the water demands to exceed the total availability, and, therefore, some users demands to be only partially satisfied, since the water allocation will be done following certain and predetermined priorities.

In any case, the discrepancy between the allocated quantity and the demand should be penalised, in such a way that an extra ‘cost’ term is included in the objective function, caused by the water shortage for a certain user at a certain time period. The penalties

should reflect in some way the losses caused by the water shortage and must be time varying, since the impacts of the water shortage are not all the times the same for a user. In addition, environmental considerations should also be taken into account in the water allocation. The simplest way to achieve that is to assign high costs in the most unsustainable water supply methods.

## 4. Mathematical Model Development

### 4.1. System parameters and variables

The variables and the parameters of the system are shown in Tables 1 and 2 respectively. The optimal planning problem will be solved in a predetermined time horizon. The length of the time horizon depends on the specific problem and the desired use of the results. Actually, the length of the time horizon will also indicate the time interval that will be the basic step for the optimisation model.

Parameter	Magnitude
$B_{jt}$	Benefit for the use of the water from user j at time interval t (in $\text{€m}^3$ )
$D_{jt}$	Demand of water from user j at time interval t ( $\text{m}^3$ )
$Q_{jt}^{\text{MIN}}$	Minimum water flow to user j at time interval t
$S_{it}$	Capacity of Supply source i ( $\text{m}^3$ ) at time interval t
$P_{jt}$	Penalty for not satisfying the demand of user j at time interval t ( $\text{€m}^3$ )
$V_{\text{max}}$	Maximum volume of water that can be stored in the storage tank ( $\text{m}^3$ )
$V_{\text{min}}$	Minimum volume of water that should be stored in the storage tank ( $\text{m}^3$ )
$C_i$	Cost of water from supply source i

Table 1: Model Parameters

Variable	Magnitude
$F_{it}$	Flow of water from supply source i at the time interval t
$Q_{jt}$	Water flow to user j at time interval t
$V_t$	Water level in the reservoir at time interval t ( $\text{m}^3$ )

Table 2: Model Variables

### 4.2. Optimisation Criterion

The optimisation criterion that will try to maximise the efficiency of the water system is the maximisation of the water value, taking into account benefits and costs, i.e.

Maximize Total Value of Water = Maximize (Total Benefit – Total Cost)

$$\text{Total Benefit} = \sum_t \sum_j B_{jt} * Q_{jt}$$

Total Cost = Supply Cost + a penalty for the discrepancy between demand and real supply to the users. Hence, the Total Cost term in the objective function is expressed as:

$$\text{Total Cost} = \sum_t \sum_i C_i * F_{it} + \sum_t \sum_j P_{jt} * (D_{jt} - Q_{jt})$$

The Benefits reflect the productivity of the water allocated to each specific user and vary with time. In fact the Benefits are determined by the area under consideration, the time of the year and the profitability of each water use. Benefits should affect the water pricing, and this is currently being applied, however not consistently and rationally. Proper quantification of Benefits is an issue of further study.

Therefore, the optimality criterion that maximises the total benefits and at the same time attempts to minimise as much as possible the costs and the differences between the quantities supplied to the users with their real requirements is expressed as follows:

$$Max \sum_t \sum_j B_{jt} * Q_{jt} - [ \sum_t \sum_i C_i * F_{it} + \sum_t \sum_j p_j * (D_{jt} - Q_{jt}) ]$$

4.3. Model Constraints

The model constraints impose limits on the problem variables and include:

The continuity equation in the water storage tank:  $V_t = V_{t-1} + \sum_i F_{it} - \sum_j Q_{jt}$

Upper and lower bounds of the water in the reservoir:  $V_{min} \leq V_t \leq V_{max}$

Capacity limitations of each supply scheme:  $F_{it} \leq S_{it}$ .

Flows allocated to each user should not exceed the corresponding Demands. Furthermore, it may be desirable to assign a minimum water quantity to some users.

$$Q_{jt}^{min} \leq Q_{jt} \leq D_{jt}$$

5. Application Results

The above model is applied in a simple case study to illustrate the type of results that can be expected of this work. A typical island of Aegean Sea is considered. The basic parameters of the problem are shown in Table 3.

Time Horizon	Twelve months, 1 month Time interval
Supply Sources	Desalination (1), ground reservoir(2), transfer by ships (3)
Users	Urban (A), Irrigation (B)
Demand Profile and Bjt	Shown in Figure 2 and Table 4 respectively
$Q_{jt}^{MIN}$	50.000 m <sup>3</sup>
$S_i$	300000, 200000 and 500000 m <sup>3</sup> /month respectively
$P_{jt}$	The same for both users, 10 €/m <sup>3</sup>
$C_i$ (cost from sources 1, 2 and 3 respectively)	$C_1= 3 \text{ €/m}^3, C_2= 4,4 \text{ €/m}^3, C_3= 7 \text{ €/m}^3$

Table 3: Data and basic assumptions for the case study

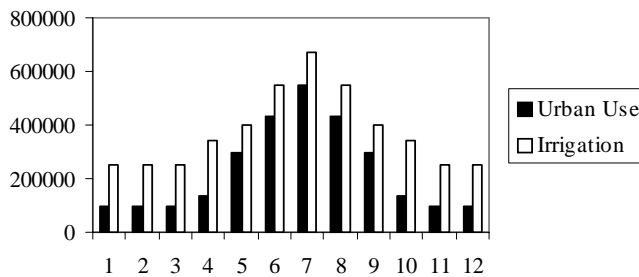


Figure 2: Demand profile for the case study

Months	1	2	3	4	5	6	7	8	9	10	11	12
$B_{At}$	5	5	5	10	15	15	15	15	10	5	5	5
$B_{Bt}$	5	5	5	5	5	5	5	5	5	5	5	5

Table 4: Benefits for the users of the Case study for twelve months (in €/m<sup>3</sup>)

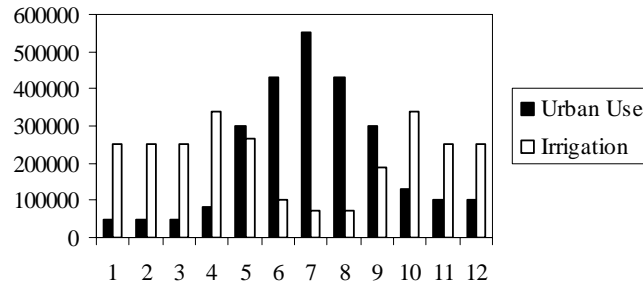


Figure 3: Model Results - Water distributed to users

In the Case Study the Benefits differ for each the two users. For the urban use the benefit increases significantly during the summer months, while it remains the same throughout the year for the irrigation use. Figure 3 shows the water distributed to each user during the year. As shown, the distribution is completely directed to the urban use during the two summer months, when the demand is the highest and the corresponding benefits for user A the biggest.

## 6. Conclusions and Further Work

An optimisation model has been proposed in order to carry out water systems planning in complex water systems with multiple supply sources and multiple users. The resulting mathematical programming model provides the water flows from each supply source and the flows allocated to each user, in order to optimise an economic / operation efficiency criterion. The model takes into account the costs of each supply source and the benefits from the water allocation to each user and allows the water availability to be less than the total demand. Thus, the work introduces the idea of optimally allocating the existing resources rather than continuously seeking ways to expand the existing sources. The present study, being part of an ongoing research, provides a basis for further formal modeling.

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