

(Reynolds Transport Theorem)

$$\left(\frac{dB}{dt} \right)_{\text{Συστ. ματος}} = \frac{d}{dt} \left(\int_{OE} \beta \rho dV \right) + \int_{EOE} \beta \rho (\vec{V}_r \cdot \vec{n}) dA$$

$$\left(\frac{dB}{dt} \right)_{\text{Συστ. ματος}} = \frac{d}{dt} \left(\int_{OE} \beta \rho dV \right) + (\beta \rho AV)_{out} - (\beta \rho AV)_{in}$$

, $m, \vec{P} = m\vec{V} \quad E,$

$\beta = \frac{B}{m} \quad \mu, \quad \beta = \frac{m}{m} = 1, \beta = \frac{\vec{P}}{m} = \vec{V}$

$$\beta = \frac{E}{m} = e = u + \frac{V^2}{2} + gz$$

$$\vec{V}_r$$

$$\vec{n} \quad \mu$$

$$\mu$$

$$dV$$

$$\left(\frac{dm}{dt} \right)_{\text{Συστ. ματος}} = 0 = \frac{d}{dt} \left(\int_{OE} \beta \rho dV \right) + \int_{EOE} \rho (\vec{V}_r \cdot \vec{n}) dA$$

$$\left(\frac{dm}{dt} \right)_{OE} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

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$$0 = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

$$- \text{USUF} (\mu \quad \mu \quad \quad \quad \& \quad \mu \quad \mu \quad \quad)$$

$$m_2 - m_1 = \sum_i m_i - \sum_e m_e$$

$$\left(\frac{d\vec{P}}{dt} \right)_{\Sigma_{\text{οστ}} \mu\alpha\tau\omicron\varsigma} = \Sigma \vec{F} = \frac{d}{dt} \left(\int_{OE} \rho \vec{V} dV \right) + \int_{EOE} \rho \vec{V} (\vec{V}_r \cdot \vec{n}) dA$$

$$\mu \quad \quad \mu \quad \quad \mu \quad (\quad \mu \quad \quad) \cdot \quad \mu$$

$$\mu \quad \quad \quad \mu \quad (\quad \quad \mu \quad \quad) \cdot$$

$$\mu \quad \quad \quad \quad \quad \mu$$

$$:$$

$$\Sigma \vec{F} - \int_{OE} \vec{a}_{rel} dm = \frac{d}{dt} \left(\int_{OE} \rho \vec{V} dV \right) + \int_{EOE} \rho \vec{V} (\vec{V}_r \cdot \vec{n}) dA$$

$$\vec{a}_{rel} = \frac{d\vec{R}}{dt} + \frac{d\vec{\Omega}}{dt} \times \vec{r} + 2\vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

$$\mu \quad \quad \quad \mu$$

$\mu \cdot$

$$\Sigma \vec{F} = \left(\frac{d\vec{P}}{dt} \right)_{OE} = \Sigma (\dot{m} \vec{V})_e - \Sigma (\dot{m} \vec{V})_i$$

$e \quad i$
 $\mu \quad \quad \cdot$

$$\left(\frac{d}{dt} \int (\vec{r} \times \vec{V}) dm \right)_{\Sigma_{\text{οστ}} \mu\alpha\tau\omicron\varsigma} = \Sigma \vec{M}_o = \frac{d}{dt} \left(\int_{OE} (\vec{r} \times \vec{V}) \rho dV \right) + \int_{EOE} (\vec{r} \times \vec{V}) \rho (\vec{V}_r \cdot \vec{n}) dA$$

$$\mu \quad \quad \mu \quad \quad \mu \quad (\quad \mu \quad \quad) \cdot \quad \mu$$

$$\mu \quad \quad \quad \mu \quad (\quad \quad \mu \quad \quad) \cdot$$

$$\mu \quad \quad \quad \quad \quad \mu$$

$$:$$

$$\Sigma (\vec{r} \times \vec{F})_o - \int_{OE} (\vec{r} \times \vec{a}_{rel}) dm = \frac{d}{dt} \left(\int_{OE} (\vec{r} \times \vec{V}) \rho dV \right) + \int_{EOE} (\vec{r} \times \vec{V}) \rho (\vec{V}_r \cdot \vec{n}) dA$$

$$\vec{a}_{rel} = \frac{d\vec{R}}{dt} + \frac{d\vec{\Omega}}{dt} \times \vec{r} + 2\vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

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$$\Sigma(\vec{r} \times \vec{F})_o = \left(\frac{dH}{dt} \right)_{OE} + \Sigma[\dot{m}(\vec{r} \times \vec{V})]_{oe} - \Sigma[\dot{m}(\vec{r} \times \vec{V})]_{oi}$$

$$\left(\frac{dE}{dt} \right)_{\Sigma \text{υστ } \mu \alpha \tau \omicron \varsigma} = \dot{Q} - \dot{W} = \frac{d}{dt} \left(\int_{OE} e \rho dV \right) + \int_{EOE} e \rho (\vec{V}_r \cdot \vec{n}) dA$$

$$\begin{aligned} \dot{Q} - \dot{W}_s - \dot{W}_{ss} &= \frac{d}{dt} \left(\int_{OE} e \rho dV \right) + \int_{EOE} \left(e + \frac{p}{\rho} \right) \rho (\vec{V}_r \cdot \vec{n}) dA \\ &= \frac{d}{dt} \left(\int_{OE} e \rho dV \right) + \int_{EOE} \left(h + \frac{V^2}{2} + gz \right) \rho (\vec{V}_r \cdot \vec{n}) dA \end{aligned}$$

\dot{W}_s

\dot{W}_{ss}

μ

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$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

$$\left(\frac{d(U + KE + PE)}{dt} \right)_{OE} = \dot{Q}_{OE} - \dot{W}_{OE} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

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μ

$$0 = \dot{Q}_{OE} - \dot{W}_{OE} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

- USUF

$$m_2 \left(h_2 + \frac{V_2^2}{2} + gz_2 \right) - m_1 \left(h_1 + \frac{V_1^2}{2} + gz_1 \right) = \dot{Q}_{OE} - \dot{W}_{OE} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

Reynolds : $Re = \frac{\rho U x_{cr}}{\mu}$ $Re = \frac{U x_{cr}}{\nu}$

() : $Re_{cr} \approx 2500$

() : $Re_{cr} \approx 5 \cdot 10^5$

von Karman

$\tau_0(x) = \rho U^2 \frac{d\theta}{dx}$ $\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy$

(. .)

$\frac{u}{U} = \alpha \left(\frac{y}{\delta}\right)^3 + \beta \left(\frac{y}{\delta}\right)^3 + \gamma \left(\frac{y}{\delta}\right)^3 + \varepsilon$

: $u(x,0) = 0, \quad \partial^2 u / \partial y^2(x,0) = 0$

$u(x,\delta) = U, \quad \partial u / \partial y(x,\delta) = 0$

$$\frac{u}{U} = -\frac{1}{2} \left(\frac{y}{\delta}\right)^3 + 1.5 \left(\frac{y}{\delta}\right)^3$$

$\frac{u}{U} = \alpha_2 \left(\frac{y}{\delta}\right)^2 + \alpha_1 \left(\frac{y}{\delta}\right) + \alpha_0$

: $u(x,0) = 0$

$u(x,\delta) = U, \quad \partial u / \partial y(x,\delta) = 0$

$$\frac{u}{U} = -\left(\frac{y}{\delta}\right)^2 + 2 \left(\frac{y}{\delta}\right)$$

$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$

μ

: $\tau_{0t}(x) = 0.0288 \rho U^2 (Re_x)^{-0.2}$

	μ	μ	μ	μ
	Blasius			
μ	$\frac{u}{U} = f\left(y\sqrt{\frac{U}{\nu x}}\right)$	$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$	$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$
μ	$\delta = \frac{5x}{\sqrt{\text{Re}_x}}$	$\delta = \frac{4,64x}{\sqrt{\text{Re}_x}}$	$\delta = \frac{5,48x}{\sqrt{\text{Re}_x}}$	$\delta = \frac{0,37x}{\text{Re}_x^{1/5}}$
μ	$\delta^* = \frac{1,73x}{\sqrt{\text{Re}_x}}$	$\delta^* = \frac{3}{8}\delta$	$\delta^* = \frac{1,827x}{\sqrt{\text{Re}_x}}$	$\delta^* = \frac{1}{8}\delta$
μ	$\theta = \frac{0,664x}{\sqrt{\text{Re}_x}}$	$\theta = \frac{39}{280}\delta$	$\theta = \frac{2}{15}\delta$	$\theta = \frac{7}{72}\delta$
	$C_f = \frac{0,664}{\sqrt{\text{Re}_x}}$	$C_f = \frac{0,646}{\sqrt{\text{Re}_x}}$	$C_f = \frac{0,728}{\sqrt{\text{Re}_x}}$	$C_f = \frac{0,058}{\text{Re}_x^{1/5}}$
	$C_D = \frac{1,328}{\sqrt{\text{Re}_L}}$	$C_D = \frac{1,292}{\sqrt{\text{Re}_L}}$	$C_D = \frac{1,456}{\sqrt{\text{Re}_L}}$	$C_D = \frac{0,074}{\text{Re}_L^{1/5}}$
μ	$\tau_0(x) = C_f \frac{\rho U^2}{2}$	$\tau_0(x) = C_f \frac{\rho U^2}{2}$	$\tau_0(x) = C_f \frac{\rho U^2}{2}$	$\tau_0(x) = C_f \frac{\rho U^2}{2}$
μ	$F_D = C_D \frac{\rho U^2}{2} A$	$F_D = C_D \frac{\rho U^2}{2} A$	$F_D = C_D \frac{\rho U^2}{2} A$	$F_D = C_D \frac{\rho U^2}{2} A$

μ

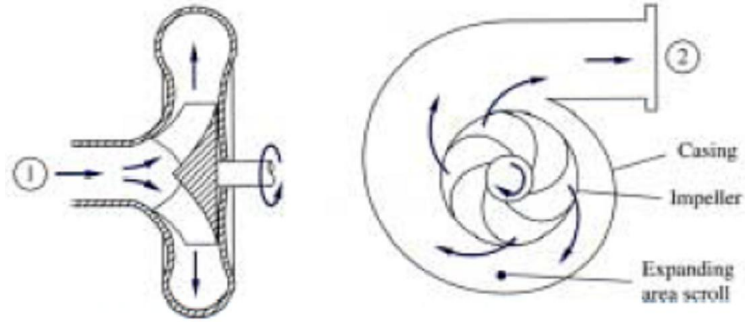
$$C_D = \frac{0,074}{\text{Re}_L^{1/5}} - \left(\frac{\text{Re}_{cr}}{\text{Re}_L}\right) \left(\frac{0,074}{\text{Re}_{cr}^{0,2}} - \frac{1,328}{\text{Re}_{cr}^{0,5}}\right) \quad 5 \times 10^5 \leq \text{Re}_L \leq 10^7$$

$$C_D = \frac{0,455}{(\log \text{Re}_L)^{2,58}} - \left(\frac{\text{Re}_{cr}}{\text{Re}_L}\right) \left(\frac{0,455}{(\log \text{Re}_L)^{2,58}} - \frac{1,328}{\text{Re}_{cr}^{0,5}}\right) \quad \text{Re}_L \leq 10^9$$

$$\text{Re}_{cr} = 5 \times 10^5$$

$$C_D = \frac{0,074}{\text{Re}_L^{1/5}} - \frac{1740}{\text{Re}_L} \quad 5 \times 10^5 \leq \text{Re}_L \leq 10^7$$

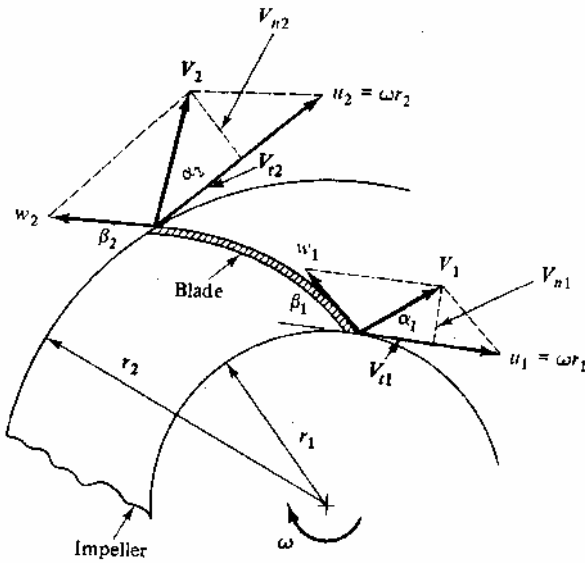
$$C_D = \frac{0,455}{(\log \text{Re}_L)^{2,58}} - \frac{1700}{\text{Re}_L} \quad \text{Re}_L \leq 10^9$$



$$H = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + Z \right)_2 - \left(\frac{p}{\rho g} + \frac{V^2}{2g} + Z \right)_1 = \frac{1}{2g} [(V_2^2 - V_1^2) + (u_2^2 - u_1^2) + (w_2^2 - w_1^2)]$$

$$P = \rho g Q H$$

$$\eta = \frac{P}{\omega T} = \frac{\rho g Q H}{\omega T}$$



$$T = \rho Q (r_2 V_{t2} - r_1 V_{t1})$$

$$P = \omega T = \rho Q (u_2 V_{t2} - u_1 V_{t1})$$

$$V_{t2} = V_2 \cos \alpha_2 = u_2 - w_2 \cos \beta_2 = u_2 - \frac{V_{n2}}{\tan \beta_2}$$

$$H = \frac{u_2 V_{t2} - u_1 V_{t1}}{g} = \frac{1}{2g} [(V_2^2 - V_1^2) + (u_2^2 - u_1^2) + (w_2^2 - w_1^2)]$$

$$\alpha_1 = 90^\circ \quad V_{t1} = 0$$

$$P = \omega T = \rho Q (u_2 V_{n2} \cot \alpha_2 - u_1 V_{n1} \cot \alpha_1)$$

$$V_{n1} = V_1 \sin \alpha_1 = w_1 \sin \beta_1 = \frac{Q}{2\pi r_1 b_1} \quad V_{n2} = V_2 \sin \alpha_2 = w_2 \sin \beta_2 = \frac{Q}{2\pi r_2 b_2}$$

$$H \approx \frac{u_2^2}{g} - \frac{u_2 \cot \beta_2}{2\pi r_2 b_2 g} Q$$

μ

$$(NPSH) \quad NPSH = \frac{P_i}{\rho g} + \frac{V_i^2}{2g} - \frac{P_v}{\rho g}$$

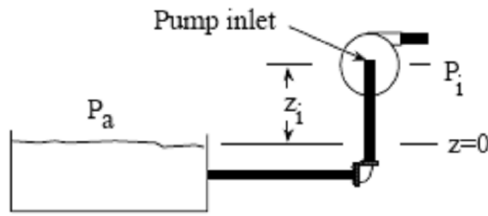
$$NPSH = \frac{P_i}{\rho g} + \frac{V_i^2}{2g} - \frac{P_v}{\rho g} = \frac{P_a}{\rho g} - Z_i - h_f - \frac{P_v}{\rho g}$$

μ

μ

$$\sigma = \frac{NPSH}{H}$$

μ



μ

μ

μ

$$C_Q = \frac{Q}{nD^3}, \quad C_H = \frac{gH}{n^2 D^2}, \quad C_P = \frac{bhp}{\rho n^3 D^5} = \frac{\omega T}{\rho n^3 D^5}, \quad Re = \frac{\rho n D^2}{\mu} \quad \frac{\varepsilon}{D}$$

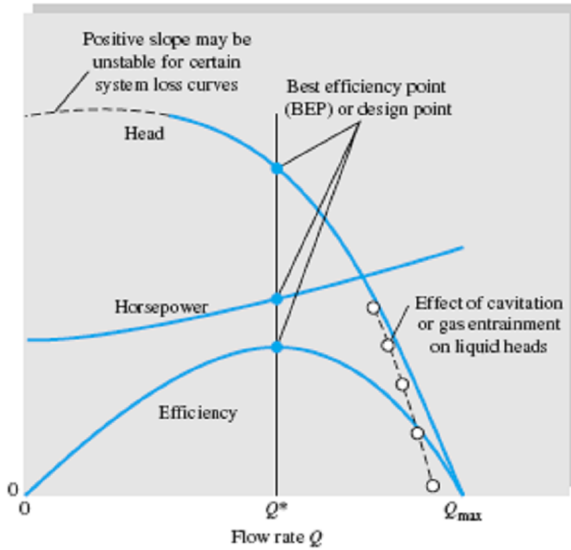
$$C_H = f\left(C_Q, Re, \frac{\varepsilon}{D}\right) \quad C_P = f\left(C_Q, Re, \frac{\varepsilon}{D}\right)$$

μ

$$C_H \approx f(C_Q) \quad C_P \approx f(C_Q)$$

$$\eta \equiv \frac{C_H C_Q}{C_P} = \eta(C_Q)$$

$$C_{HS} = \frac{g(NPSH)}{n^2 D^2} = C_{HS}(C_Q)$$



μ :

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$$(C_Q)_1 = (C_Q)_2 \Rightarrow \frac{Q_2}{Q_1} = \frac{n_2}{n_1} \left(\frac{D_2}{D_1} \right)^3$$

$$(C_H)_1 = (C_H)_2 \Rightarrow \frac{H_2}{H_1} = \left(\frac{n_2}{n_1} \right)^2 \left(\frac{D_2}{D_1} \right)^2$$

$$(C_P)_1 = (C_P)_2 \Rightarrow \frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1} \right) \left(\frac{n_2}{n_1} \right)^3 \left(\frac{D_2}{D_1} \right)^5$$

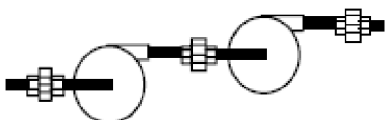
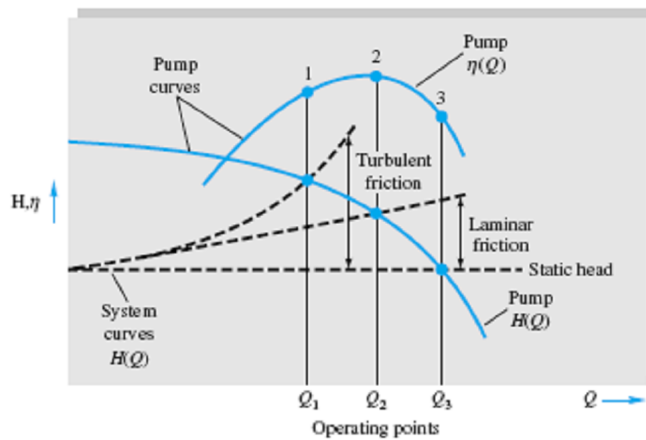
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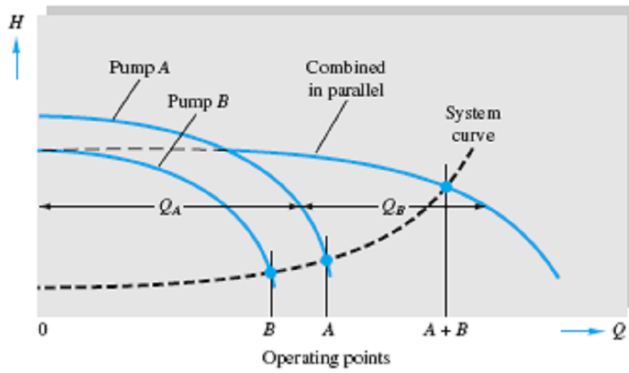
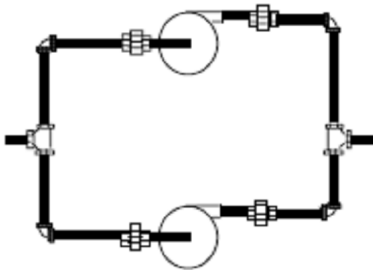
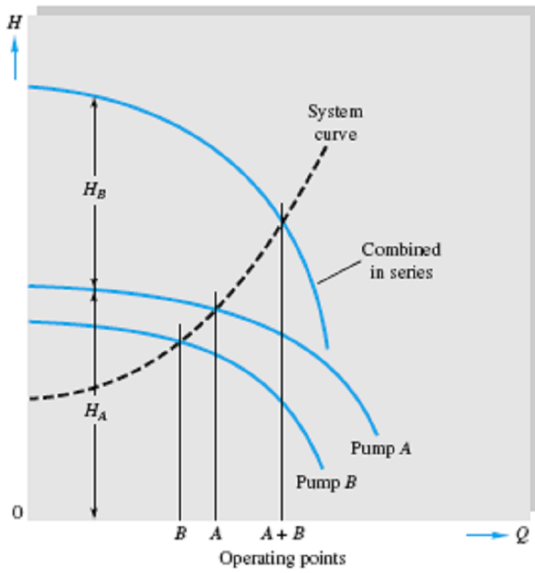
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$$N'_S = \frac{\sqrt{C_{Q^*}}}{(C_{H^*})^{3/4}} = \frac{n\sqrt{Q^*}}{(gH^*)^{3/4}}$$

$$N_S = \frac{n\sqrt{Q^*}}{(H^*)^{3/4}}$$

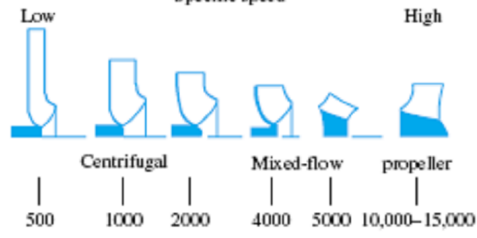
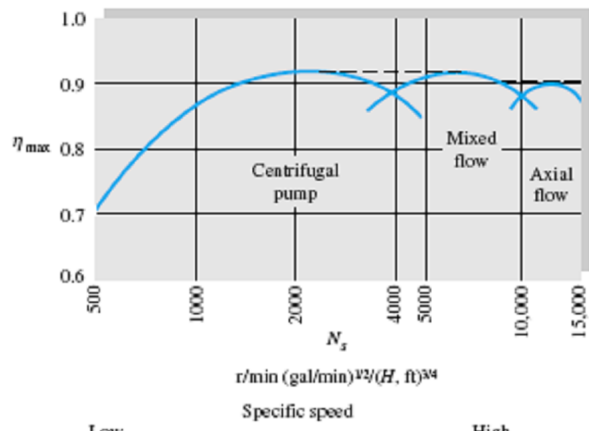
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(b)