MODULATION RECOGNITION OF DIGITAL SIGNALS USING THE WAVELET TRANSFORM

Konstantinos Maliatsos  Stavroula Vassaki  Philip Constantinou
National Technical University of Athens
Athens, Greece

ABSTRACT
This paper presents an algorithm for modulation recognition of received signals in the presence of additive white Gaussian noise (AWGN) with the use of the wavelet transform. The decision is made based on the extraction of some special features of the Continuous Wavelet Transform of the received signal. The Haar wavelet was used as the mother wavelet. Besides the SNR, the algorithm needs no further information for the received signal such as signal bandwidth or carrier frequency. It is able to classify PSK, QAM, FSK and ASK signals, as well as to identify the modulation order. Effort was made in order to reduce the computational workload, so that the algorithm can be suitable for real-time recognition. After extended simulation the algorithm proved to be practically inerrable for SNR>12dB and achieves low rates of false detection for SNR=10dB.

I. INTRODUCTION
The Software Defined Radio (SDR) concept [1,2] suggests that future radio systems will support multi-standard, multimode and multiband wireless communications. A function that could provide valuable service to a fully adaptive and reconfigurable SDR transceiver is the automatic signal classification and modulation recognition. These functions will simplify the design process of a multimode communication system. Over the years, many studies have been conducted regarding these subjects. The evolution of signal processors and analogue to digital converters allow the identification of more modulation types with greater accuracy and less computational load. The developed algorithm was used for classification of M-FSK, M-PSK, M-ASK (M ∈ {2, 4, 8}) and M-QAM (M ∈ {8, 16, 32, 64}) signals. M=2^m, where m is the number of bits per symbol. Besides the SNR, no other information for the received signal, such as the signal bandwidth or carrier frequency, is necessary.

II. BACKGROUND AND SYSTEM MODEL
A wavelet is a properly chosen, zero-mean waveform of finite duration. The wavelet transform analyzes the signal into scaled and time-shifted versions of the original wavelet (called mother wavelet), as described by:

\[ CWT(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t)\psi^*(\frac{t-b}{a})dt, a \neq 0 \]  \hspace{1cm} (1)

Eq. (1) is the expression of the continuous wavelet transform. Scaling factor \( a \) compresses or expands the wavelet giving duration. The wavelet transform analyzes the signal into scaled and time-shifted versions of the original wavelet (called mother wavelet), as described by:

\[ \frac{1}{\sqrt{|a|}}\psi^\left(\frac{k}{a}\right) = \begin{cases} 
\frac{1}{\sqrt{|a|}}a^{-1/2}k < a/2 \\
\frac{1}{\sqrt{|a|}}a^{-1}\left(\frac{1}{2} - k\right), 0 < k < a/2 \\
0, \text{otherwise}
\end{cases} \] \hspace{1cm} (2)

The received AWGN waveform was modelled by the equation \( r(t) = x(t) + n(t) \) where \( n(t) \) is additive white noise and \( x(t) \) is the information signal, described by:

\[ s(k) = \sqrt{5} \sum_{i=1}^{M} \left( A_i + j B_i \right) e^{j(k-\omega_i)t}\nu_i(t) \] \hspace{1cm} (3)

For M-FSK modulation \( \omega_i \in \{\omega_1, \omega_2, \ldots, \omega_M\}, A_i=1, B_i=0 \) and \( \theta_i \) the initial phase.
For M-PSK modulation \( \omega_i \) is the carrier frequency, \( A_i=1, B_i=0 \) and \( \theta_i \) ∈ \{initial phase+\( \pi(m-1)/M \), \( m=1,2,\ldots,M \} \).
For M-ASK modulation \( \omega_0 \) is the carrier frequency, \( A_1=1, B_i=0 \) and \( \theta \) the initial phase.
For M-QAM modulation \( \omega_0 \) is the carrier frequency, \( A_i \in \{2m-1-m, \ m=1,2,\ldots,m_1\}, B_i \in \{2m-1-m, \ m=1,2,\ldots,m_2\} \) (\( M=2^{m_1+m_2} \)) and \( \theta \) the initial phase. \( S \) is a constant factor that is used for adjustment of signal power, \( T_S \) is the sampling period and \( T_D \) is the duration of a modulation symbol.

The representation in (3) describes the signal after the analogue to digital conversion stage, although there is still a carrier frequency. This is in respect with the SDR principles that suggests signal digitization in RF or initial IF stages. Moreover the exact carrier frequency is unknown. Although the signal is discrete, the algorithm uses the continuous wavelet transform. This was chosen because the discrete

\[ s(k) = \sqrt{5} \sum_{i=1}^{M} \left( A_i + j B_i \right) e^{j(k-\omega_i)t}\nu_i(t) \] \hspace{1cm} (3)

For M-FSK modulation \( \omega_i \in \{\omega_1, \omega_2, \ldots, \omega_M\}, A_i=1, B_i=0 \) and \( \theta_i \) the initial phase.
For M-PSK modulation \( \omega_i \) is the carrier frequency, \( A_i=1, B_i=0 \) and \( \theta_i \) ∈ \{initial phase+\( \pi(m-1)/M \), \( m=1,2,\ldots,M \} \).
For M-ASK modulation \( \omega_0 \) is the carrier frequency, \( A_1=1, B_i=0 \) and \( \theta \) the initial phase.
For M-QAM modulation \( \omega_0 \) is the carrier frequency, \( A_i \in \{2m-1-m, \ m=1,2,\ldots,m_1\}, B_i \in \{2m-1-m, \ m=1,2,\ldots,m_2\} \) (\( M=2^{m_1+m_2} \)) and \( \theta \) the initial phase. \( S \) is a constant factor that is used for adjustment of signal power, \( T_S \) is the sampling period and \( T_D \) is the duration of a modulation symbol.

The representation in (3) describes the signal after the analogue to digital conversion stage, although there is still a carrier frequency. This is in respect with the SDR principles that suggests signal digitization in RF or initial IF stages. Moreover the exact carrier frequency is unknown. Although the signal is discrete, the algorithm uses the continuous wavelet transform. This was chosen because the discrete
transform increases the noise and distortion level (due to aliasing) in each stage of analysis [11] causing degradation of results. It must be noticed that in the proposed algorithm the wavelet transform is used for signal feature extraction, so there is no signal reconstruction stage that could eliminate aliasing effects. The continuous transform for a discrete signal can be defined by:

\[ C_{a,b} = \frac{1}{\sqrt{a}} \sum_{k} s(k) \left( \int_{-\infty}^{+\infty} \psi^*(\frac{t-b}{a}) \, dt \right) \]  

(4)

assuming that \( s(t) = s(k) \) for \( t \in [k, k+1] \)

Some technical issues that concern this algorithm are the following: The signal is oversampled in comparison with the Nyquist rate (typically \( F_c > 10F_0 \)). Finally due to the fact that the algorithm relies on the synthesis of a histogram, low pass smoothing of the histogram may help in automatic recognition procedures.

III. INTERCLASS RECOGNITION

This algorithm uses the results of the CWT to classify the signals in respect with the modulation type. Using equations (2, 3, 4) it is attempted to compute the CWT of a PSK signal. During a pulse, signal phase remains constant. In this case:

\[ CWT_{PSK}(a,nT) = \frac{S}{\alpha} \sum_{k=0}^{a-1} \exp(j(\omega_o(k+n)T_i + \theta_i + \phi_i)) - \sum_{k=0}^{a/2-1} \exp(j(\omega_o(k+n)T_i + \theta_i + \phi_i)) \]

\[ = \frac{S}{\alpha} \exp(j(\omega_oT_i + \theta_i + \phi_i)) \left( \sum_{k=0}^{a/2-1} \exp(j(\omega_o(k-\alpha)/2)T_i)) - \sum_{k=0}^{a/2-1} \exp(j(\omega_o(k)T_i)) \right) \]

\[ = \frac{S}{\alpha} \exp(j(\omega_oT_i + \theta_i + \phi_i)) \cdot \frac{1}{\exp(j(\omega_o/2)T_i)} \left( \sum_{k=0}^{a/2-1} \exp(j(\omega_oT_i)) - 1 \right) \sum_{k=0}^{a/2-1} \exp(j(\omega_oT_i)) \]

(5)

where \( \theta_0 \) is the random initial phase and \( \alpha/2 \), the number of samples included in the time scale \( \alpha \). As seen the absolute value of the above result is not depending on \( \alpha \). Also the normalization of the result with \( \sqrt{T_i} \) provides independency from sampling rate. Moreover, the sum of exponentials reminds the DFT of a sampled ideal pulse of \( \alpha/2 \) sample duration. Finally:

\[ |CWT_{PSK}(a,n)| = \frac{S}{\alpha} \left| \sin^2(\omega_o'a/4) \right| \sin(\omega_o/2) \]

(6)

for the duration of a single pulse.

Similar analysis for the other modulations results in the following equations:

\[ |CWT_{ASK}(a,n)| = \frac{S}{\alpha} \left| \sin^2(\omega_o'a/4) \right| \sin(\omega_o/2) \]

(7)

\[ |CWT_{QAM}(a,n)| = \frac{S}{\alpha} \left| \sin^2(\omega_o'a/4) \right| \sin(\omega_o/2) \]

(8)

where \( S = \sqrt{A^2 + B^2} \)

In PSK and QAM signals when the received symbol changes, a phase shift \( \delta \), is noticed. When a phase transition happens, the above equations are not valid. Instead a noticeable peak appears. Assuming that the symbol change happens at the sample \( n+\delta \), the following apply for the PSK case:

\[ C_{a,b} = \frac{S}{\alpha} \exp(j(\omega_o(n+\delta)T + \theta_i + \phi_i)) \sum_{k=0}^{a/2-1} \exp(j(\omega_o(kT)+\theta_i)) + \exp(j\delta) \sum_{k=0}^{a/2-1} \exp(j(\omega_o(kT)+\theta_i)) \]

(10)

Where \( c_\alpha \) = -1 for \( -\alpha/2 < k < 0 \) and 1 for \( 0 < k < \alpha/2 \) as the result of the Haar integral (2, 4). A careful look at (10) can lead to the following observations. First of all, the term \( \exp(j\delta) \) will definitely change the outcome in comparison with (5). Secondly assuming \( u \) is an integer \( \in -a'/2 : a'/2-1 \), the outcome of the transform diverges from (6) for \( \alpha-1 \) steps. For \( u=0 \) the value of the transform will be referred as main peak. The absolute values of the transform for \( u=0 \) are symmetrical due to the symmetry of the Haar mother wavelet. In the following study after extended tests it was decided that for \( \alpha'=4 \) samples the best performance is achieved. In this case the outcome for \( u=0 \) is the main peak and the outcome for \( u=1 \) is called secondary peaks. Similar conclusions are derived for QAM signals. Other modulations with no phase shift do not appear peaks at the transition from a symbol to another.

These results are not sufficient to provide the necessary criteria for interclass modulation recognition, but further manipulation is needed. As described in [8] the first measure is to calculate \( CWT_{\text{norm}} \) of the normalized signal by its amplitude \( (s(t)=s(t)/\|s(t)\|) \). Finally the quantity \( |\text{HWT}| \) is determined by applying a median filter in the amplitude of CWT. This filter cuts off the peaks that appear during a phase transition. Now, interclass classification can be conducted following the rules below:

- CWT (no normalization) of PSK signals is described by a single dc level and some peaks that appear in phase transitions. CWTs of FSK, QAM and ASK signals result in multi-step functions. Peaks also appear in QAM phase transition (Figures 3, 4, 5, 8).

- CWT\(_{\text{norm}}\) with normalization will not affect constant envelope modulations (PSK, FSK). However the multi-step result for QAM, ASK modulation will now be replaced by a DC level (and peaks for the QAM case)

First (Figure 1) \( |\text{CWT}| \) is passed through the median filter in order to cut off peaks. The standard deviation \( \sigma \) of the resulted \( |\text{HWT}| \) will be ideally zero for a PSK signal. On the contrary, it should be quite larger than zero for all the other modulations due to the multi-step nature. Then, \( |\text{CWT}_{\text{norm}}| \) is computed and passed through the filter. Similarly the standard deviation for ASK and QAM signals will be ideally zero. \( |\text{HWT}_{\text{norm}}| \) for FSK signals retains the multi-step amplitude for multi-step functions.
variations. In order to separate ASK and QAM signals, the $|CWT|_{norm}$ is used. Standard deviation of $|CWT|_{norm}$ for ASK is ideally zero, because no information is transmitted through phase shifts. On the other hand $|CWT|_{norm}$ for QAM will present some peaks when a phase transition occurs. Thus standard deviation is expected greater.

However, during transmission noise distortion is always present. Thus, for AWGN channels, in any case the standard deviation of the estimated transforms is not zero. Consequently, proper threshold values must be defined in order to separate the “ideally zero” and the non-zero deviations. It can easily be shown by (4) that the standard deviation of the CWT for a frame of zero-mean complex Gaussian random variables is equal with the standard deviation $\sigma$ of the Gaussian process. Given that $\sigma^2$ is also the average noise power, it is clear that the threshold values are SNR depended.

After extended simulation, two optimum threshold values were found. The first one separates the signals in proportion to the $|HWT|$ magnitude and the second one separates them in proportion to the $|HWT|_{norm}$ magnitude. If the variance of the signal is smaller than the values of the thresholds then the signal is recognised as PSK. On the contrary, if the variance of the signal is always bigger than the values of the thresholds, the signal is classified as FSK. Finally if the variance of $|HWT|$ is greater than the first threshold but the variance of $|HWT|_{norm}$ is smaller than the second then the signal is either QAM or ASK.

A similar procedure, that includes the definition of a third threshold, is followed for QAM-ASK classification via $|CWT|_{norm}$ deviation (Fig 2).

The three thresholds that are used in the decision process can be easily approximated by polynomials. This is presented in Table 1.

<table>
<thead>
<tr>
<th>SNR (in dB)</th>
<th>THRESHOLD 1</th>
<th>THRESHOLD 2</th>
<th>THRESHOLD 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.000</td>
<td>1.9e-10</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>-6.2e-11</td>
<td>-1.29e-8</td>
<td>1.3e-10</td>
</tr>
<tr>
<td>6</td>
<td>7.58e-9</td>
<td>4.94e-7</td>
<td>-2.07e-8</td>
</tr>
<tr>
<td>5</td>
<td>-3.9e-7</td>
<td>-1.18e-5</td>
<td>1.34e-6</td>
</tr>
<tr>
<td>4</td>
<td>1.1e-5</td>
<td>1.84e-4</td>
<td>-4.57e-5</td>
</tr>
<tr>
<td>3</td>
<td>-1.88e-4</td>
<td>-1.86e-3</td>
<td>8.64e-4</td>
</tr>
<tr>
<td>2</td>
<td>1.99e-3</td>
<td>1.75e-2</td>
<td>-8.51e-3</td>
</tr>
<tr>
<td>1</td>
<td>-1.3e-2</td>
<td>-4.29e-2</td>
<td>3.13e-2</td>
</tr>
</tbody>
</table>

Table 1: Polynomial approximations of thresholds

IV. INTRACLASS RECOGNITION

- **M-FSK**
  It is clear from (7) that as an ideal case, $|CWT|$ of M-FSK signal is a multi-step function with M different dc levels caused by the frequency variation. Consequently, the order of modulation can easily be found by the number of peaks which appear in the histogram of $|CWT|$. If $M/2 + 1$ to $M$ peaks appear, the input is identified as M-FSK.

- **M-ASK**
  It is observed from (9) that $|CWT|$ of an M-ASK signal is also a multi-step function (because of the amplitude variation) with M different dc levels. The same procedure as in M-FSK can be followed. If $M/2 + 1$ to $M$ peaks appear in the histogram of the $|CWT|$ magnitude of the signal, the input is identified as M-ASK.
M-PSK

Figure 5 shows that |CWT| of a PSK signal is constant. However, peaks appear when a phase change occurs. Two different procedures are followed in order to recognize M-PSK signals, depending on SNR.

- M-PSK recognition for SNR > 22 dB

In this case, peaks in |CWT| are quite large and the peak to noise ratio is high. It is proved in [9] that the peaks follow the Ricean distribution. For low SNR, Ricean distribution can be approximated by a Gaussian. Assuming equal possibility of phase shifts for MPSK, the probability density function (pdf) is a sum of (M-1) Gaussian variables:

\[ f_p(p) = \frac{1}{M-1} \sum_{i=1}^{M-1} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(p-p_m)^2}{2\sigma^2}\right) \]  

Random variable \( p \) denotes the peak value and \( p_m \) are the unknown mean peak values. If \( M/2 \) to \( M-1 \) Gaussians exist in the histogram of peaks, the input is identified as M-PSK [9].

- M-PSK recognition for SNR < 22 dB

When SNR < 22 dB, peaks from phase changes cannot be distinguished from noise. In order to eliminate the impact of noise, an estimation of symbol’s period is necessary. We form the autocorrelation of |CWT| magnitude:

\[ R(a', l) = \frac{1}{N} \sum_{n=0}^{N-1} |CWT_p(a', n) - |CWT_r(a')| \]  

where \( N \) is the total number of samples and \( |CWT_r(a')| \) is the sample average of the |HWT| magnitude. The autocorrelation function for white Gaussian noise is given by \( R_e(\tau) = (\pi/2)\delta(\tau) \). Autocorrelation of |CWT| (Fig 6) is described by the peak which corresponds to noise and periodical peaks will correspond to the periodical alterations of the symbols. The distance between the peaks is equal to the symbol’s period. Thus, an estimation of the symbol’s period from the histogram of the distances between the peaks is possible. This is done in order to exclude the noisy samples and keep only the |CWT| points that correspond to phase changes.

As a next step, two figures were created for PSK signals with the use of (10). The first presents the value of the main peak (Section I) in respect with the carrier frequency for each modulation order whereas the second presents the values of secondary peaks in respect with the carrier frequency.

Taking advantage of the described synchronization and symbol rate estimation procedure, the mean value of |CWT| in areas between the peaks was found. Considering that this value corresponds to zero degree phase change, it is possible to find an approximation for the carrier frequency from Figure 7, as well as the values of the possible peaks for every other order of modulation.

From [9], the |CWT| at phase change has a pdf equal to the sum of \( M \) Ricean functions.

However, if the carrier frequency is equal to the half of the sampling frequency, the primary peaks which occur due to phase changes may overlap. As a result, the order of the modulation cannot be identified just by computing the pdf of the primary peaks. Since the secondary peaks don’t appear to have similar problem for the specific carrier frequency, a new joint pdf for main and secondary peaks for every possible order (\( M=2, M=4, M=8 \)) is specified.

\[ f_p(p, q) = \frac{1}{M} \sum_{i=1}^{M-1} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(p-p_m)^2}{2\sigma^2}\right) \exp\left(-\frac{(q-p_m)^2}{2\sigma^2}\right) \]  

Finally, the identification task is to compute the likelihood value for all possible \( M \) symbols. The M that yields the largest \( \Lambda_M \) determines the decision for the modulation order of the M-PSK signal.

In order to constrain the computational complexity of this function, an approach of the modified Bessel function of the first kind and zero order is used:

\[ l_1(x) = \frac{1}{\sqrt{2\pi x}} \exp(x) \]  

M-QAM

As seen, |CWT|norm of a QAM signal is similar to the |CWT| of a PSK signal of unitary amplitude. Thus the mean value of |HWT| can be used to find the carrier frequency for
peak of zero degree phase change from Fig 7. It is easy to prove that each QAM signal of different order has different maximum $|HWT|$ amplitude. A table is created in order to correspond carrier frequency with the maximum amplitude of the multi-step $|HWT|$ of M-QAM signals. A comparison of the maximum amplitude of $|HWT|$ of the received signal with the maximum amplitudes of the table allows the detection of the order of the modulation.

![Figure 8: $|CWT|$, $|CWT|_{norm}$, $|HWT|$ and $|HWT|_{norm}$ of QAM-8 signal](image)

### V. RESULTS

The presented algorithm gave exceptional results for SNR $\geq$ 12 dB. After extended simulation (more than two million iterations), no classification error was noticed. This indicates a classification error rate less than $10^{-6}$. The number of symbols used for each decision (synthesis of histograms, autocorrelation estimation etc) was 100. As SNR values reduced, the error rates became significant. In the Tables (2) and (3) we present the probabilities for correct and false classification for SNR=10 dB.

Table (2) shows the probabilities for interclass recognition. As input the modulation types with the most unstable (worst case) behaviour were considered. As seen for SNR=10dB there is small probability of false classification of PSK modulation as QAM.

<table>
<thead>
<tr>
<th>input/output</th>
<th>PSK</th>
<th>FSK</th>
<th>QAM</th>
<th>ASK</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSK-8</td>
<td>98.6%</td>
<td>0%</td>
<td>1.4%</td>
<td>0%</td>
</tr>
<tr>
<td>FSK-2</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>QAM-8</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>ASK-2</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 2: Results for interclass recognition

In Table (3) the results of the simulation for intraclass recognition (SNR=10dB) are presented. Generally the rate of success was more than 98%. Here it must be underlined that the results are quite better than those presented in [9].

<table>
<thead>
<tr>
<th>M-PSK (in/out)</th>
<th>PSK-2</th>
<th>PSK-4</th>
<th>PSK-8</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSK-2</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>PSK-4</td>
<td>0.1%</td>
<td>99.9%</td>
<td>0%</td>
</tr>
<tr>
<td>PSK-8</td>
<td>0%</td>
<td>0.2%</td>
<td>99.8%</td>
</tr>
<tr>
<td>M-FSK (in/out)</td>
<td>FSK-2</td>
<td>FSK-4</td>
<td>FSK-8</td>
</tr>
<tr>
<td>FSK-2</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>FSK-4</td>
<td>0.2%</td>
<td>98.3%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>

Table 3: Results for intraclass recognition

### VI. CONCLUSIONS

The algorithm presents many modifications and improvements in comparison with other wavelet based modulation recognizers:

- It is able to classify QAM and ASK modulations. Intraclass recognition for QAM/ASK was also successful.
- The algorithm is able to detect carrier frequency for PSK, QAM and ASK modulations.
- The use of continuous wavelet transform gave better results, as no alias distortion was introduced.
- Proper thresholds were identified and presented.
- The procedure for PSK (snr<22dB) improved significantly the results.
- The overall performance of the algorithm is better.

### REFERENCES


